3.4 Videos Guide

3.4a

- Infinite limits vs. limits at infinity

Definition: (limit at infinity)

- Let f be a function defined on some interval (a, ∞). Then $\lim_{x\to\infty} f(x) = L$ means that for every $\varepsilon > 0$ there is a corresponding number N such that if $x > N$ then $|f(x) - L| < \varepsilon$. (The definition for a limit at negative infinity is analogous.)

Definition: (horizontal asymptote)

- The line $y = L$ is called a horizontal asymptote of the curve $y = f(x)$ if either $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$.

Theorem (statement):

- If $r > 0$ is a rational number, then $\lim_{x\to\infty} \frac{1}{x^r} = 0$. If $r > 0$ is a rational number such that x^r is defined for all x, then $\lim_{x\to-\infty} \frac{1}{x^r} = 0$.

Exercises:

- Find the limit or show that it does not exist.
  - $\lim_{x\to\infty} \frac{4x^3+6x^2-2}{2x^3-4x+5}$
  - $\lim_{x\to-\infty} \frac{4x^3+6x^2-2}{2x^3-4x+5}$

3.4b

  - $\lim_{x\to\infty} \left(\sqrt{4x^2+3x} + 2x\right)$
  - $\lim_{x\to-\infty} \left(\sqrt{4x^2+3x} + 2x\right)$

3.4c

  - $\lim_{x\to\infty} \frac{\sqrt{4x^2+1}}{3x-1}$
  - $\lim_{x\to-\infty} \frac{\sqrt{4x^2+1}}{3x-1}$